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# CYCLIC FLUCTUATIONS OF WATER LEVEL AS A BASIS FOR DETERMINING AQUIFER TRANSMISSIBILITY

By
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This contribution was orginally prepared as a technical paper from the United States for presentation in August 1951 at the Brussels Assembly of the International Union of Geodesy and Geophysics. Al-though it will ultimately appear in the Assembly proceedings, as published by the International Association of Hydrology, the paper is considered of sufficient interest to warrant interim release and distribution by the Geological Survey.

In coastal areas, wells near bodies of tidal water frequently exhibit sinusoidal fluctuations of water level, in response to periodic changes of tidewater stage. Inland, the regulation of a surface reservoir often produces correlative changes of ground-water stage in wells adjacent either to the reservoir or to its attendant stream. As the stage of the surface water rises, the head upon the subaqueous outcrop of the aquifer increases and thereby either increases the rate of inflow to the aquifer or reduces the rate of outflow therefrom. The increase in recharge or reduction in discharge results in a general recovery of water level in the aquifer. On the subsequent falling stage this pattern is reversed. When the stage of the surface body fluctuates as a simple harmonic motion a train of sinusoidal waves is propagated shoreward through the sub-outcrop of the aquifer. With increasing distance from the sub-outcrop, the amplitude of the transmitted wave decreases and the time lag of a given maximum or minimum increases.

If there is no suboutcrop, but the aquifer is confined by an extensive aquiclude, the rise and fall of the surface-water stage changes the total weight upon the aquifer. Resultant variations in compressive stress are borne in part by the skeletal aquifer and in part by its confined water. The relative compressibilities of the skeletal mass and the confined water determine the ratio of stress assignment and the net response of the piezometric surface to the surface force.

The problem of potential distribution within a semi-infinite solid, with the face at $x = 0$, normal to the infinite dimension and subjected to periodic variations of potential, was long ago analyzed and the solution employed by Angstrom (Carslaw, pp. 41-44) to determine the thermal conductivity of various solids. Similar analyses have been used by other investigators to determine the conductivity of the earth, the penetration of diurnal and annual temperature waves, and the flow of heat in the walls of a steam-engine cylinder. The physical nature of these problems is quite analogous to our problem of the aquifer having a suboutcrop under tidewater or a regulated surface stream. Consequently, these solutions provide a ready pattern for evaluating the hydrologic counterpart.

Assume a homogeneous aquifer of uniform thickness and of great areal extent shoreward - that is, normal to the strike of the suboutcrop. Assume also that water is released immediately with a decline in pressure and at a rate proportional to that decline. As a further simplification, assume that flow is unidimensional and that the aquifer is fully penetrated by the surface-water body that propagates the cyclic fluctuations. In those situations where the aquifer is not fully penetrated or where it is under water-table (unconfined) conditions the analysis will still be satisfactory if (1) the observation well is far enough from the suboutcrop so that it is unaffected by vertical components of flow and (2) the range in cyclic fluctuation at the observation well is only a small fraction of the saturated thickness of the formation. The fundamental differential equation for the linear flow of water in an aquifer intersected by a stream may be written as follows (Ferris, 1950, p. 286):

$$\frac{\partial^2 s}{\partial x^2} = \frac{S}{T}\frac{\partial s}{\partial t} \qquad (1)$$

in terminology adapted to this problem

- s = net rise or fall of ground-water stage with reference to the mean stage over an observed period.
- x = distance from suboutcrop to observation well.
- t = time elapsed from convenient reference node within any cycle.
- S = coefficient of storage
- T = coefficient of transmissibility

Let $s_o$ designate the amplitude or half range of stage fluctuation of the surface body. The problem resolves then to finding the particular solution of equation (1) that will satisfy the following boundary condition:

$$s = s_o \sin\omega t \qquad \text{at } x = o \tag{2}$$

The mathematical development leading to the particular solution of the differential equation is given in considerable detail by Ingersoll, Zobel, and Ingersoll (1948, pp. 46-47) and only the final form is repeated herewith, as

$$s = s_o e^{-x\sqrt{\omega S/2T}} \sin\left(\omega t - x\sqrt{\frac{\omega S}{2T}}\right) \tag{3}$$

If we designate the period of the uniform tide or stage by $t_o$ in accord with Jacob (1950, p. 365) then $\omega$ may be expressed in radians per time unit as $2\pi/t_o$ and equation (3) becomes

$$s = s_o e^{-x\sqrt{\pi S/t_o T}} \sin\left(\frac{2\pi t}{t_o} - x\sqrt{\frac{\pi S}{t_o T}}\right) \tag{4}$$

Equation (4) defines a wave motion whose amplitude rapidly decreases with distance x as given by the factor $s_o e^{-x\sqrt{\pi S/t_o T}}$. When the aquifer response is due to loading rather than head change at the suboutcrop the amplitude factor should be reduced (Jacob, 1950, p. 356) by the ratio $[\alpha/(\alpha + \theta\beta)]$, where $\alpha$ is the vertical compressibility of the skeletal aquifer, $\beta$ is the compressibility of the water, and $\theta$ is the porosity of the sand. While values of $\beta$, the compressibility of water can be obtained from published tables of physical data, little information is available to estimate $\alpha$, the vertical compressibility of the aquifer and this factor may vary considerably. The principal guide to the magnitude of $\alpha$ would be correlative data from similar aquifers where pumping test results have established its value.

From equation (4) the range of ground-water fluctuation at an observation well at a distance x from the suboutcrop is given by

$$s_r = 2\, s_o e^{-x\sqrt{\pi S/t_o T}} \tag{5}$$

It is of interest to note from the form of equation (5) that the slower the fluctuation of the surface tide - that is, the greater the value of $t_o$, the greater is the range of stage within the aquifer.

Let $t_l$ denote the lag in time of occurrence of a given maximum or minimum ground-water stage following the occurrence of a similar surface stage. Then, from Ingersoll, Zobel, and Ingersoll (1948, p. 48), the expression for $t_l$ is as follows:

$$t_l = x\sqrt{\frac{t_o S}{4\pi T}} \tag{6}$$

The apparent velocity of transmission of the wave through the aquifer is

$$v_{ap} = \frac{x}{t_l} = \sqrt{\frac{4\pi T}{t_o S}} \tag{7}$$

Equation (7) indicates the apparent velocity of a given maximum or minimum and does not pertain either to the rate of pressure transmission (Muskat, 1939, p. 669) or the apparent rate of pressure transmission (Jacob, 1940, p. 585) within the aquifer.

The wave length is obtained as

$$\lambda = v_{ap}\ t_o = \sqrt{\frac{4\pi t_o T}{S}} \tag{8}$$

Physically there would be little opportunity in ground-water hydrology to obtain a snapshot view of the sinusoidal wave train, as would be required to employ equation (8).

During half the cycle water flows through the suboutcrop into the aquifer; in the other half it flows out again. The quantity of flow per half cycle is determined with the aid of Darcy's law.

$$Q = TIL$$

where L is the length of suboutcrop across which the flow occurs, and where I, as given by Ingersoll, Zobel, and Ingersoll (1948, p. 49), is:

$$I = \frac{\partial s}{\partial x} = s_o e^{-x\sqrt{\omega S/2T}}\left(-\frac{\omega S}{2T}\right)\left[\sin\left(\omega t - x\sqrt{\frac{\omega S}{2T}}\right) + \cos\left(\omega t - x\sqrt{\frac{\omega S}{2T}}\right)\right] \tag{9}$$

It is convenient to set up the integral for the quantity of flow per unit length of suboutcrop. Because the gradient $\partial s/\partial x$ is not in phase with s, the limits of integration are determined by noting from inspection of equation (9) that at $x = 0$ the gradient is zero at $t = -\pi/4\omega = -t_o/8$, reaches a minimum at $t = \pi/4\omega = t_o/8$, and returns to 0 at $t = 3\pi/4\omega = 3t_o/8$.

$$\frac{Q}{L} = -T \int_{-t_o/8}^{3t_o/8} \left(\frac{\partial s}{\partial x}\right)_{x=o} dt \tag{10}$$

$$\frac{Q}{L} = -T \int_{-\pi/4\omega}^{3\pi/4\omega} \left(\frac{\partial s}{\partial x}\right)_{x=o} dt \tag{11}$$

$$\frac{Q}{L} = s_o \sqrt{\frac{2t_oST}{\pi}} \tag{12}$$

To illustrate the applicability of these methods to field problems, data are presented for three riverside observation wells at the Ashland station of the municipally owned water supply of the City of Lincoln, Nebraska. A map of the well station is shown as figure 1. Automatic water-stage recorders are operated on observation wells 1 to 3, inclusive, and at the river gage on the Platte River at the crossing of U. S. Highway 6. A geologic section from west to east through supply well 2 is reproduced as figure 2. Typical records from the autographic charts for the river-stage recorder and observation well 1 are reproduced as figure 3.

Observation wells 1, 2, and 3 are respectively 42, 106, and 252 feet from the edge of the river at a normal stage. Each well is screened and developed in the upper part of the aquifer. From records for the river-stage recorder and the three observation wells for the period September 23 to 29, 1950, the ratio of ground-water fluctuation to river change was computed for the rising and falling limb of each cycle. These stage ratios are summarized in table 1. The period of the river fluctuation, computed for each limb of each cycle, ranged from

9 1 1 2 5 0 1 3 0 2



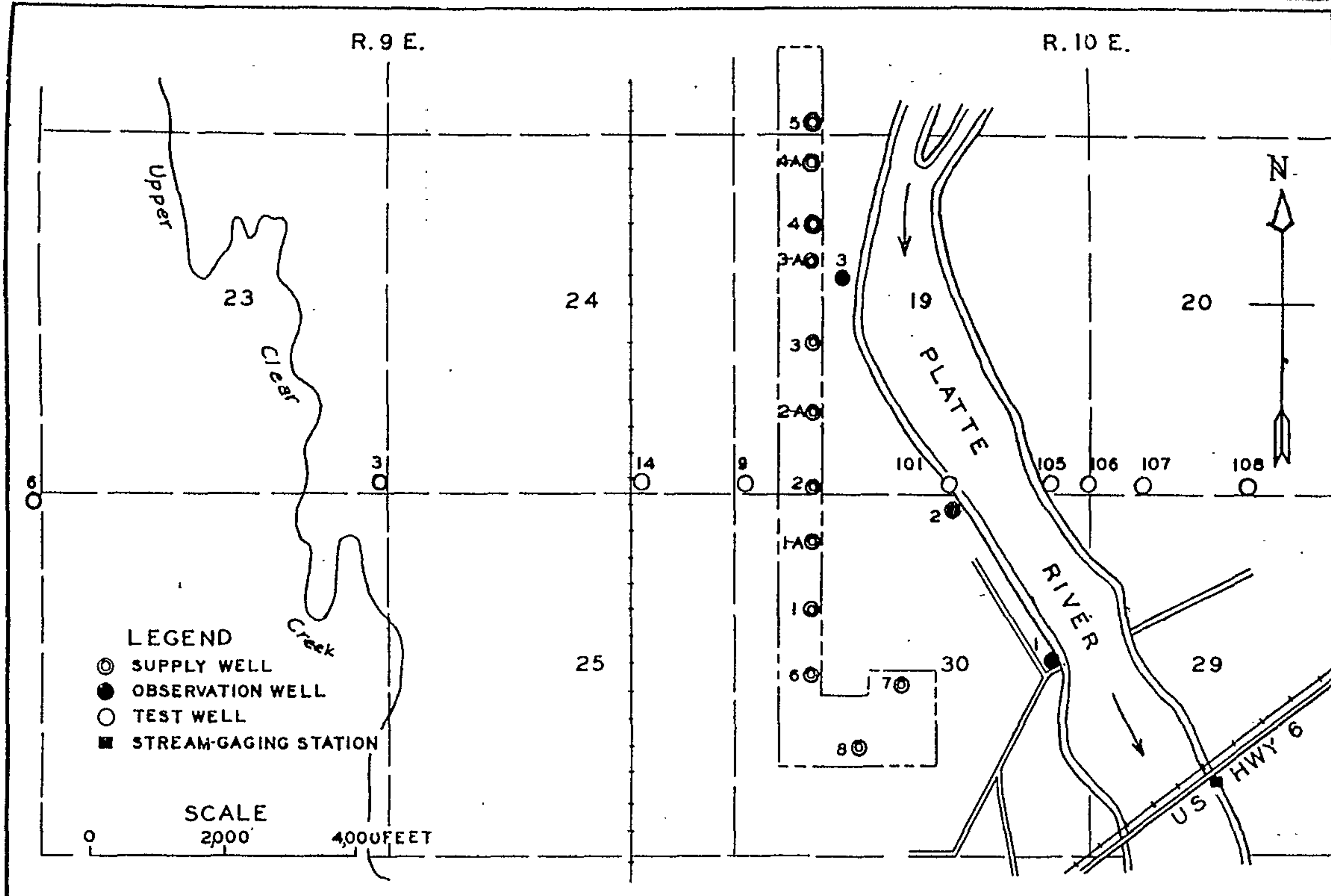


FIGURE I.-MAP SHOWING A PORTION OF THE CITY OF LINCOLN WELL STATION AT ASHLAND, NEBR.

9 1 1 2 4 5 0 1 3 ? 3

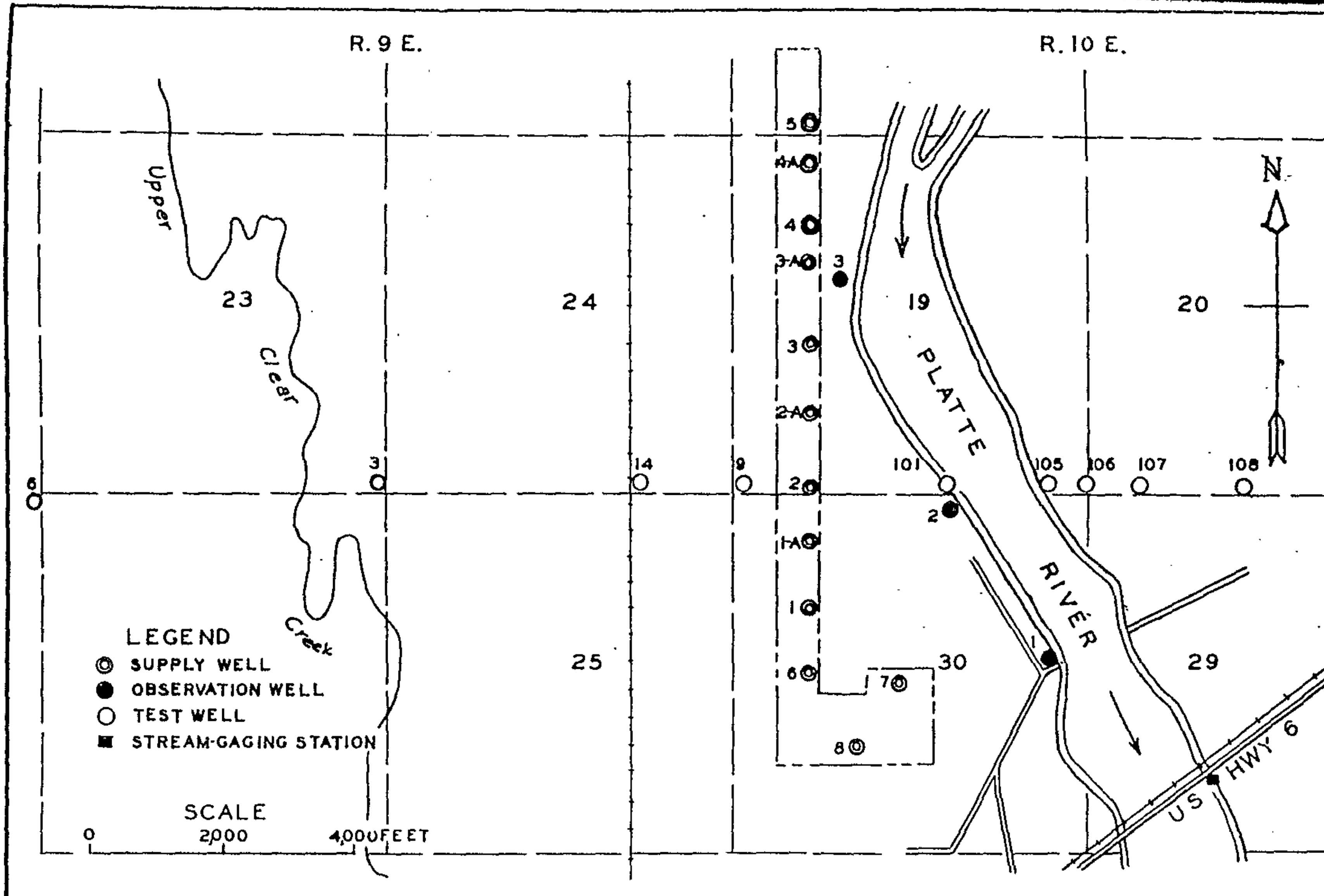


FIGURE 1.-MAP SHOWING A PORTION OF THE CITY OF LINCOLN WELL STATION AT ASHLAND, NEBR.

9 1 1 4 5 7 1 3 7 4

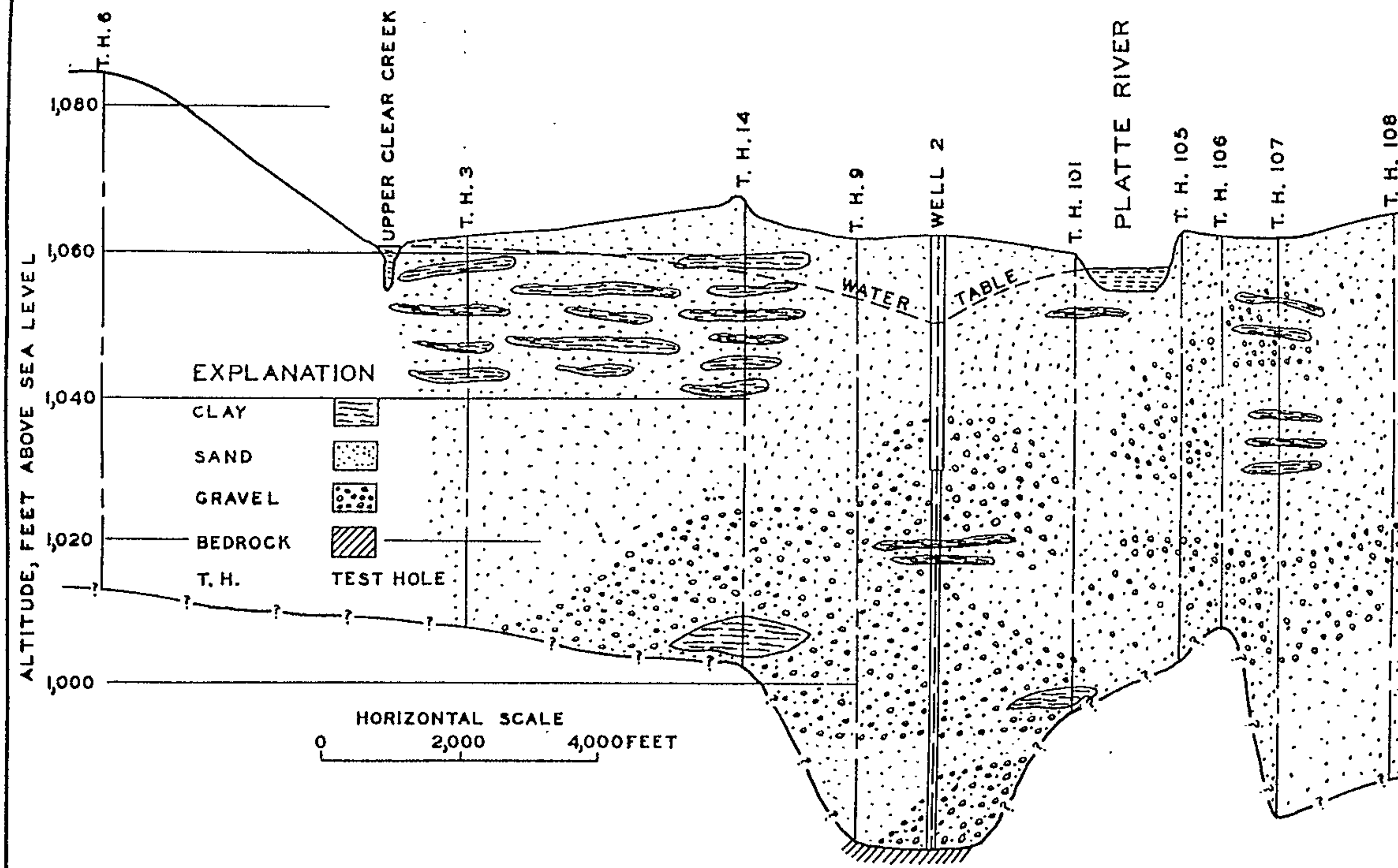


FIGURE 2.- GENERALIZED GEOLOGIC SECTION FROM WEST TO EAST THROUGH SUPPLY WELL 2 OF THE CITY OF LINCOLN AT THE ASHLAND, NEBR. WELL STATION.

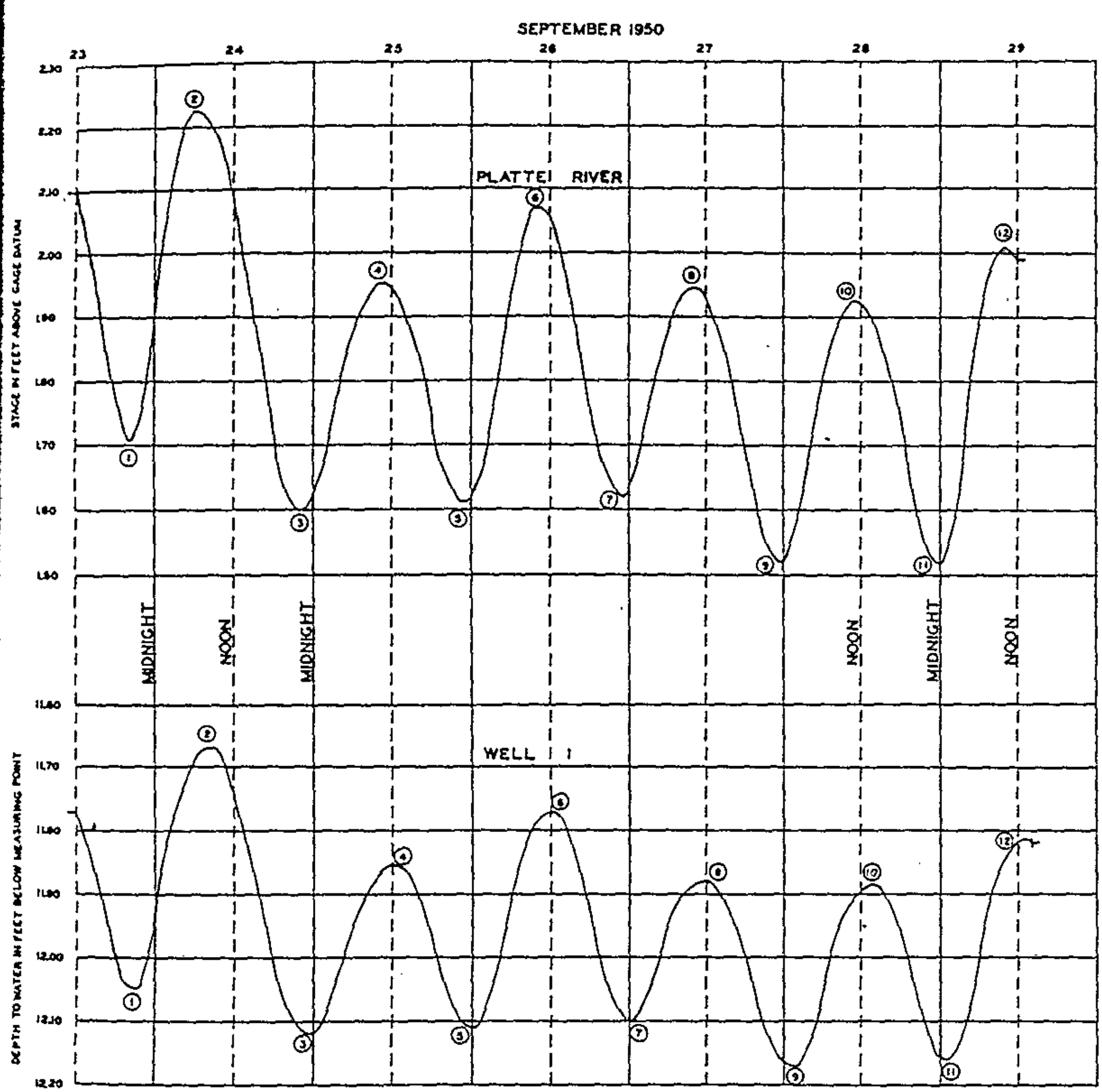


FIGURE 3.-GRAPH SHOWING THE STAGE OF THE PLATTE RIVER AND THE WATER LEVEL IN OBSERVATION WELL 1 OF THE CITY OF LINCOLN AT THE ASHLAND, NEBR. WELL STATION.

Table 1. Ratio of the recorded range of ground-water stage in observation wells 1, 2, and 3 as compared to the stage range of the Platte River at Ashland, Nebr.

| Reference numbers of fluctuation a/ | Well 1 | | Well 2 | | Well 3 | |
|---|---|---|---|---|---|---|
| | Rising stage | Falling stage | Rising stage | Falling stage | Rising stage | Falling stage |
| 1-2 | 0.73 | | 0.52 | | 0.35 | |
| 2-3 | | 0.71 | | 0.56 | | 0.46 |
| 3-4 | .77 | | .54 | | .31 | |
| 4-5 | | .76 | | .56 | | .29 |
| 5-6 | .74 | | .59 | | .26 | |
| 6-7 | | .73 | | .56 | | .29 |
| 7-8 | .69 | | .47 | | .28 | |
| 8-9 | | .71 | | .56 | | .20 |
| 9-10 | .72 | | .52 | | .33 | |
| 10-11 | | .68 | | .51 | | .37 |
| 11-12 | .71 | | .53 | | .14 | |
| Averages | .73 | .72 | .53 | .55 | .28 | .32 |
| | 0.72 | | 0.54 | | 0.30 | |

a/ See figure 3.

5 hours to 31.0 hours and averaged 24 hours or 1 day. To apply the ge-ratio data, equation (5) is adapted for use with gallon-per-day--foot units, as follows:

$$s_r = 2s_o e^{-4.8x\sqrt{S/t_oT}} \qquad (13)$$

$$\frac{s_r}{2s_o} = e^{-4.8x\sqrt{S/t_oT}}$$

$$\log_{10}\left(\frac{s_r}{2s_o}\right) = -2.1x\sqrt{\frac{S}{t_oT}} \qquad (14)$$

$$2.1\sqrt{\frac{S}{t_oT}} = \frac{-\log_{10}\left(s_r/2s_o\right)}{x} \qquad (15)$$

The logarithmic quantity $(s_r/2s_o)$ is in effect the ratio of the ange of ground-water stage to the range of river or tide stage. The orm of equation (15) suggests the use of a semilogarithmic plot of the ogarithm of the range ratio versus the distance x for each observation well. Thus the right-hand member of equation (15) represents the slope of this plot, and if the change in logarithm of the range ratio is selected over one log cycle then the numerator of this slope expression reduces to unity. Thus equation (15) becomes

$$2.1\sqrt{\frac{S}{t_oT}} = -\frac{1}{\Delta x} \qquad (16)$$

A more convenient form is gained by removing the radical

$$4.4\left(\frac{S}{t_oT}\right) = \frac{1}{\Delta x^2}$$

$$T = \frac{4.4\Delta x^2}{t_o} S \qquad (17)$$

As indicated by equation (17) it is necessary that S, the coefficient of storage, be known in order to evaluate T, the coefficient of transmissibility. However, reasonable estimates of S can be made if it is known whether the aquifer is locally artesian or nonartesian which generally can be determined from studies of well logs and water-level records.

From the stage-ratio data of table I, a semilogarithmic plot, figure 4, was constructed. Using the $\Delta x$ value indicated for one log cycle, $t_o$ = 1 day, and assuming various S values for water-table conditions, as suggested by the section of figure 2, equation (17) yields the following:

$$T = \frac{4.4 \cdot (540)^2 S}{1} = 1,300,000 \ S$$

$$T = 130,000 \quad \text{if} \quad S = 0.10$$

$$T = 190,000 \qquad S = 0.15$$

$$T = 260,000 \qquad S = 0.20$$

$$T = 320,000 \qquad S = 0.25$$

At the suboutcrop where x = o, the range of the aquifer response, $s_r$, is equal to the river or tidal range $2s_o$, or $s_r/2s_o$ approaches unity as x approaches zero. Thus, the negative x value noted on figure 4 at the range ratio of unity represents the effective distance off shore to the suboutcrop.

The withdrawal of ground water from the numerous municipal wells nearby is relatively steady, except for minor changes in rate and distribution of pumping. These changes are more apt to disturb the time of maxima or minima observed than the ratio of well-to-river change. Further, the compressed time scale of the water-level recorders limits interpolation for this purpose to a greater degree than does the gage-height scale. Any variations in the effective screen resistance of each observation well would also tend to distort observations of maxima or minima timing. The wide range in the observed lag of maxima and minima shown by table 2 may result from any one or a combination of several of the aforementioned causes. To apply the time-lag data, equation (6) is modified as follows:

$$t_1 = \frac{x}{2}\sqrt{\frac{St_o}{\pi T}}$$

$$t_1^2 = \frac{x^2 St_o}{4\pi T}$$

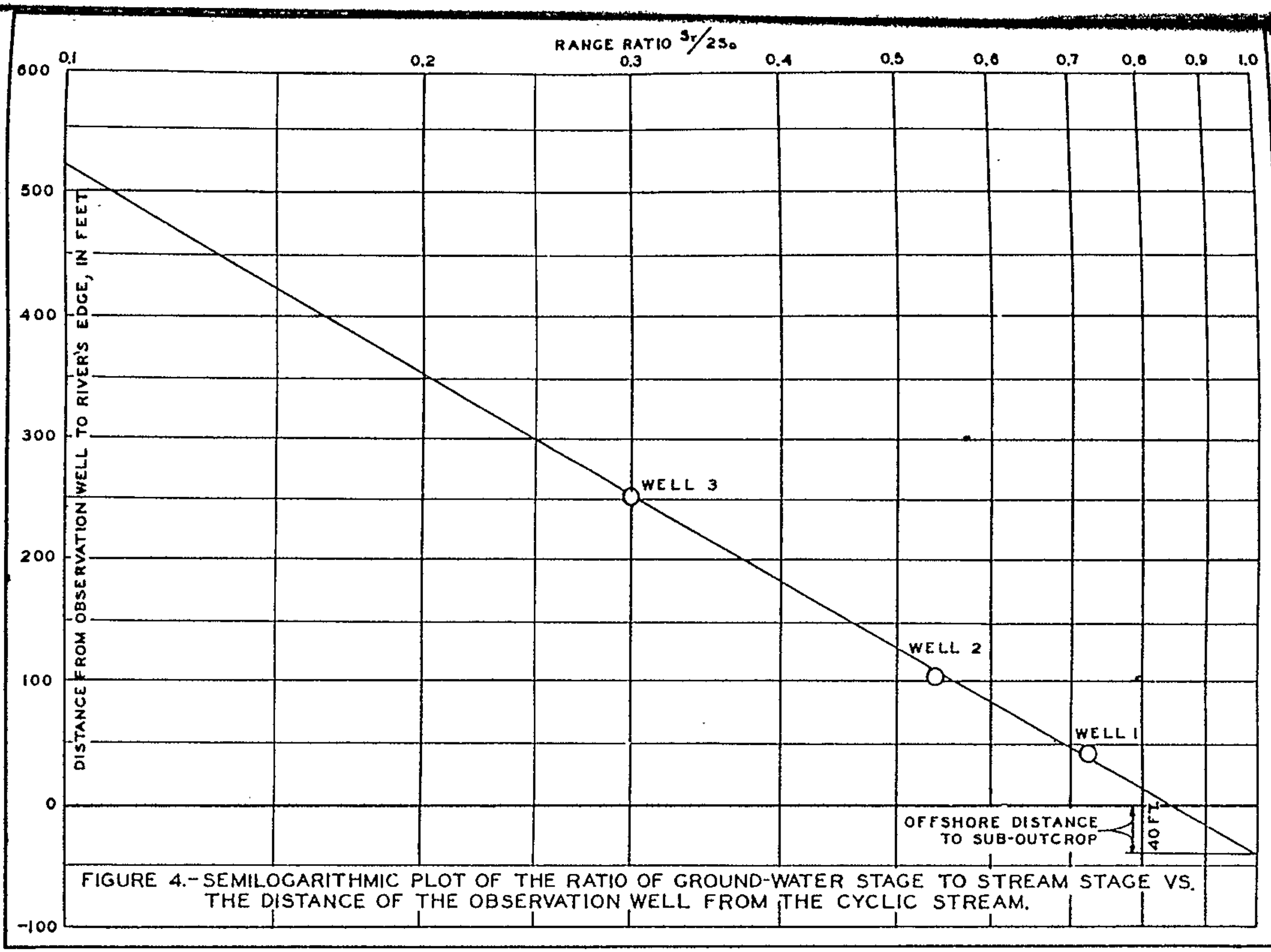


FIGURE 4.- SEMILOGARITHMIC PLOT OF THE RATIO OF GROUND-WATER STAGE TO STREAM STAGE VS. THE DISTANCE OF THE OBSERVATION WELL FROM THE CYCLIC STREAM.

Table 2. Time lag, in hours, of minimum or maximum ground-water stage relative to Platte River stage, as recorded by observation wells 1, 2, and 3 at the Ashland well station of the City of Lincoln, Nebr.

| Reference number of fluctuation a/ | Well 1 | | Well 2 | | Well 3 | |
|---|---|---|---|---|---|---|
| | Minimum | Maximum | Minimum | Maximum | Minimum | Maximum |
| 1 | 1.25 | | 3.75 | | -- | |
| 2 | | 2.50 | | 3.50 | | 6.00 |
| 3 | 2.00 | | 4.00 | | 7.50 | |
| 4 | | 2.25 | | 2.75 | | 6.75 |
| 5 | 1.75 | | 3.75 | | 6.75 | |
| 6 | | 2.25 | | 3.25 | | 5.75 |
| 7 | 1.50 | | 4.00 | | 6.50 | |
| 8 | | 2.00 | | 2.50 | | 5.50 |
| 9 | 2.50 | | 4.00 | | 7.00 | |
| 10 | | 2.75 | | 2.25 | | 6.75 |
| 11 | 2.25 | | 3.75 | | 6.75 | |
| 12 | | 2.50 | | 2.50 | | 5.50 |
| Averages | 1.90 | 2.40 | 3.90 | 2.80 | 6.70 | 6.00 |
| | 2.1 | | 3.3 | | 6.3 | |

a/ See figure 3.

$$T = \frac{x^2 S t_o}{4\pi t^2} \tag{18}$$

For T in gallons per day per foot and $t_o$ and $t_1$ in days, equation (18) becomes

$$T = \frac{0.60\ x^2 S t_o}{t_1^2} \tag{19}$$

The average values of $t_1$, the time lag, are plotted for each well in figure 5. The slope of this graph is $x/t_1$ which appears in equation (19) at the square exponent. Substituting in equation (19) the slope coordinates noted on figure 5 there results

$$T = \frac{0.60 \times \overline{250}^2 \times 1}{(5/24)^2}\, S$$

$$T = 860{,}000\ S$$

$$T = 86{,}000 \quad \text{if} \quad S = 0.10$$

$$T = 130{,}000 \qquad S = 0.15$$

$$T = 170{,}000 \qquad S = 0.20$$

$$T = 220{,}000 \qquad S = 0.25$$

At the suboutcrop, where $x = 0$, the time lag, $t_1 = 0$. The negative value of x indicated by figure 5 at the $t_1 = 0$ axis is the effective distance off shore to the suboutcrop.

The large difference between the coefficients of transmissibility indicated by the stage-ratio method as compared to the time-lag method may reflect the influence of the nearby pumping in distorting the gage height and time of each maximum or minimum. The T estimates obtained by both methods and resultant averages are summarized in table 3.

Considerable refinement of the T estimates would be possible by expanding the time scale on the water-stage recorders through the use of daily time gears in lieu of the normal weekly gears. It would also be desirable to make the studies during periods when the rate of withdrawal by the nearby supply wells is constant. A more adequate test of these methods might be possible with a profile line of observation wells

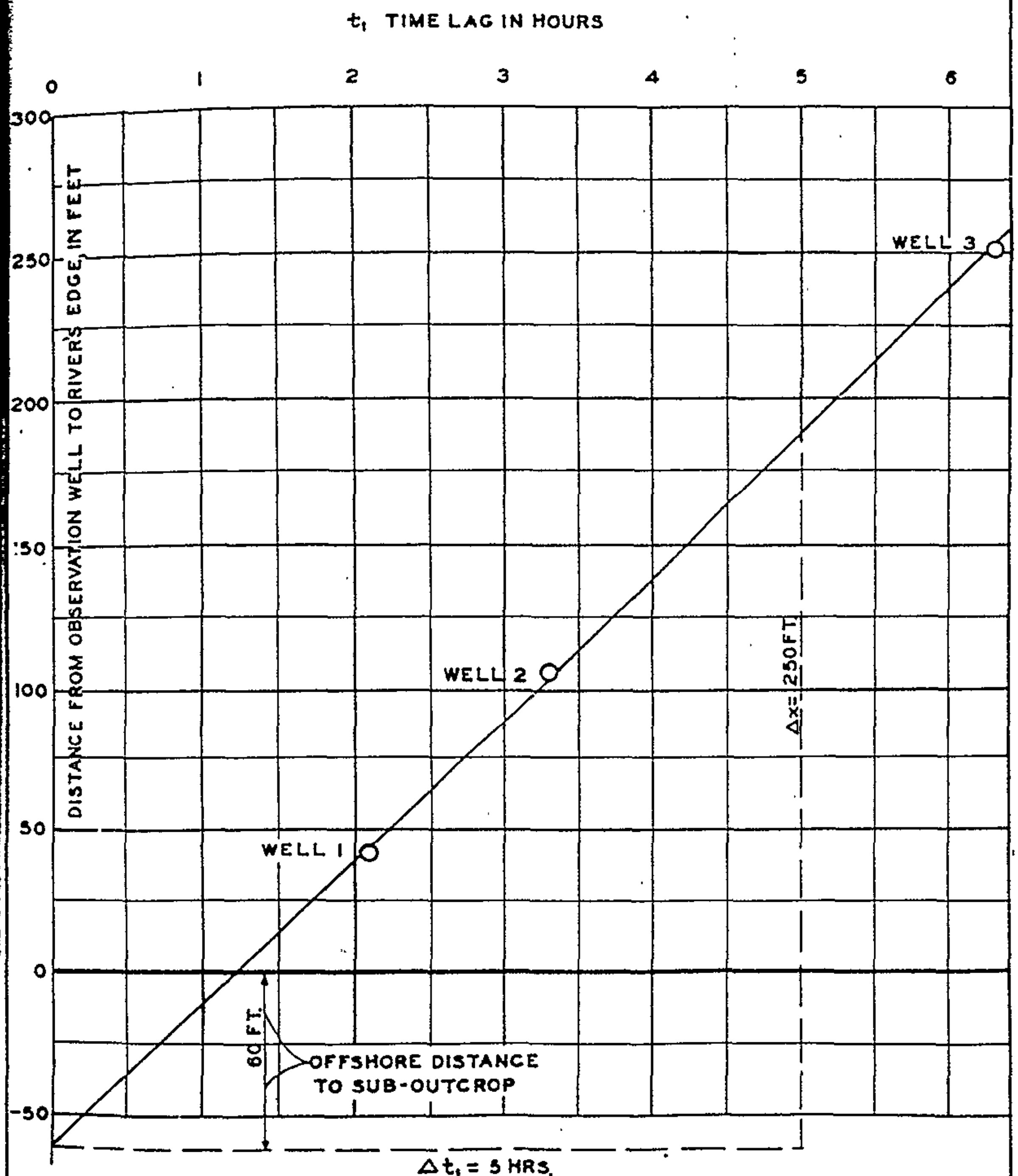


FIGURE 5.-PLOT OF TIME LAG VS. DISTANCE OF OBSERVATION WELL FROM SUB-OUTCROP.

Table 3. Summary of determinations of the coefficient of transmissibility by stage-ratio and time-lag methods.

| Method | T, coefficient of transmissibility, in gallons per day per foot | | | |
|---|---|---|---|---|
| Coefficient of storage | S=0.10 | S=0.15 | S=0.20 | S=0.25 |
| Stage-ratio method | 130,000 | 190,000 | 260,000 | 320,000 |
| Time-lag method | 86,000 | 130,000 | 170,000 | 220,000 |
| Average | 110,000 | 160,000 | 220,000 | 270,000 |

at right angles to the edge of the fluctuating surface-water body and in an area remote from heavy pumping.

It is reported that the saturated thickness of alluvial deposits in the well field area averages about 70 feet. Using this thickness and the average T values of table 3, the average permeability in gallons per day per square foot is 1,600 for S=0.10; 2,300 for S=0.15; 3,100 for S=0.20; and 3,900 for S=0.25. In comparison, gradient studies by the city, based on water-table contour maps, indicate an average permeability of 2,200 gallons per day per square foot.

The above-indicated values of the coefficients of transmissibility and storage should be considered as tentative, pending the completion of other hydrologic studies in the Ashland area. However, these data serve to demonstrate the applicability and usefulness of the methods described for analyzing cyclic fluctuations of ground-water level. Although their greatest use will be in areas of tidal streams and seas or near regulated streams and lakes, it was shown by Rambaut (1901, p. 235) that these methods can also be applied with fair results to variations that resemble periodic motion but are limited in duration to a single maximum or minimum. Thus, the response of an aquifer to the passage of a flood crest in a hydraulically connected stream may lend itself to this analysis. A logical extension from this generalized problem of a simple sinusoidal motion would be to study the applicability of the unit functions or delta functions of electrical-network analysis to the response of aquifers to complex patterns of recharge from precipitation or to the response of a stream to various rainfall-runoff patterns.

## Acknowledgments

The author is greatly indebted to H. A. Waite, district geologist of the U. S. Geological Survey at Lincoln, Nebr., for his assistance in assembling the basic data for this report and to D. L. Erickson, Director of Parks, Public Property and Improvements and City Engineer of the City of Lincoln, Nebr., and members of his staff for heir hearty cooperation and considerable effort in making these data vailable.

## uthors note

Before preparation of this report was completed, an excellent report on cyclic fluctuations of ground-water level was published by Werner and Noren (1951, pp. 238-244). The work of the author, though of subsequent date, represents an independent analysis made without prior knowledge or advance notice of their coincidental research.

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